

ACCELERATED MATHEMATICS

CHAPTER 9

GEOMETRIC PROPERTIES

PART II

TOPICS COVERED:

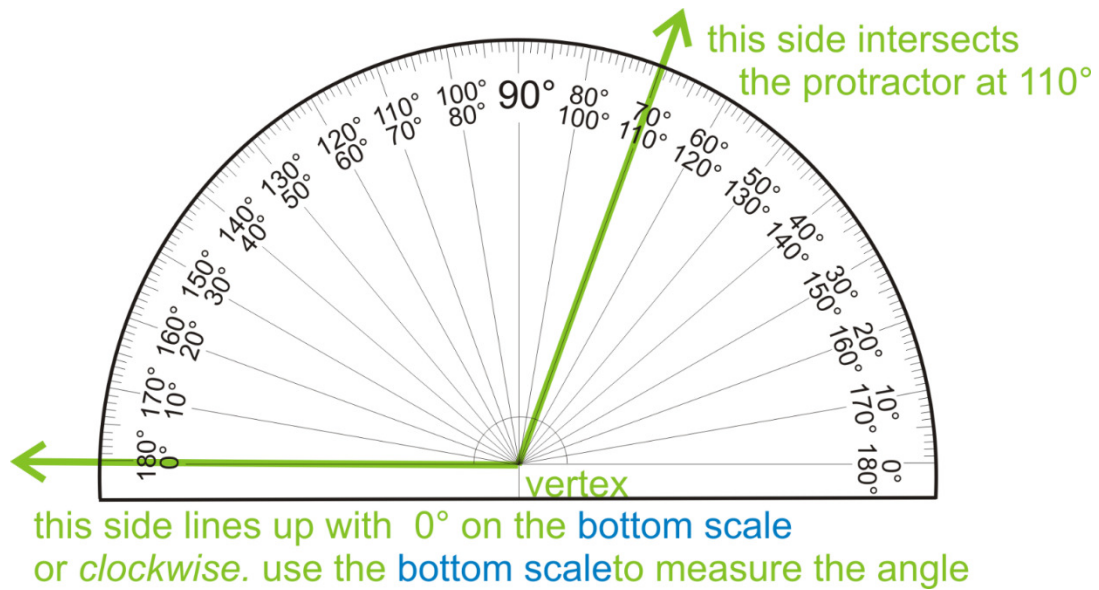
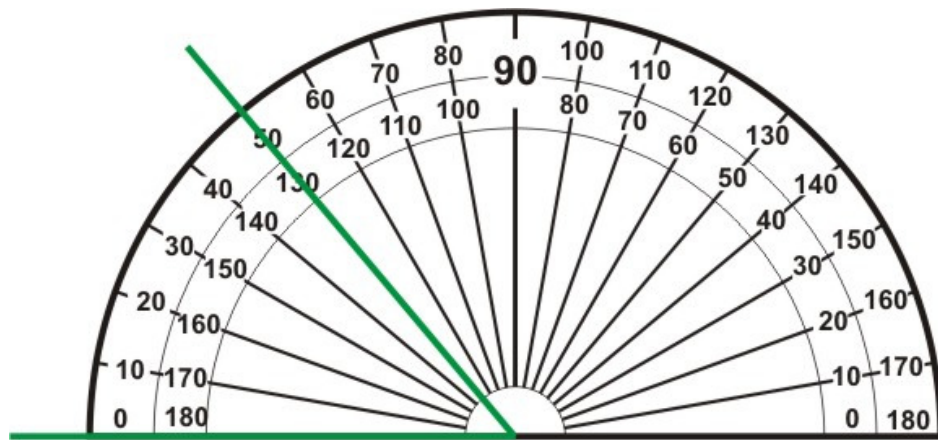
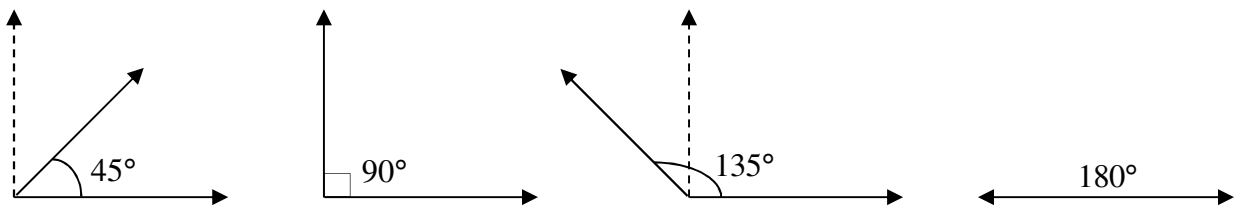
- Measuring angles
- Complementary and supplementary angles
- Triangles (sides, angles, and side-angle relationships)
- Angle relationships with transversals



PARALLEL LINES
HAVE GOT
SO MUCH IN
COMMON.
IT'S A SHAME
THEY'LL
NEVER MEET

Geometry is the area of mathematics that deals with the properties of points, lines, surfaces, and solids. It is derived from the Greek “geometra” which literally means earth measurement.

Reference Angles:

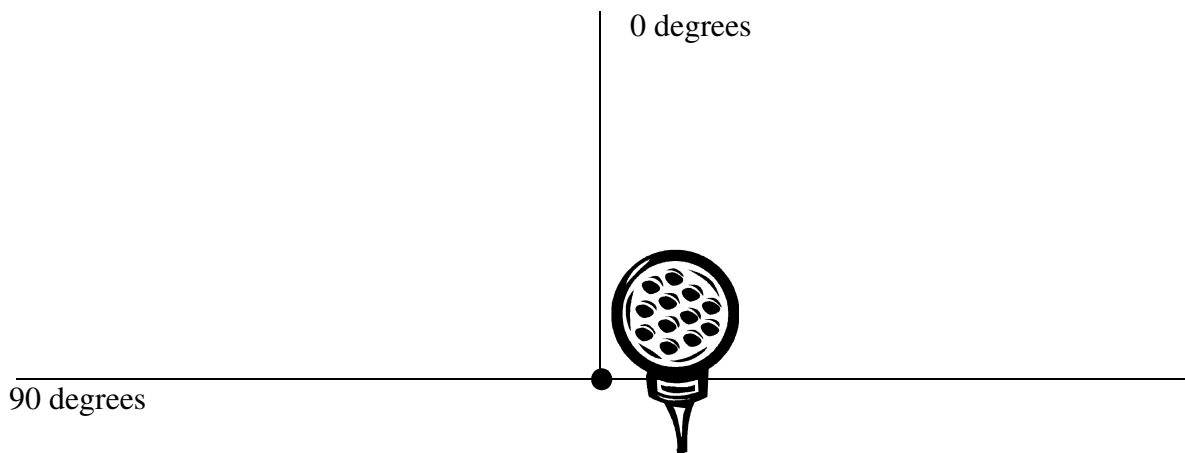


Draw the following angles using a protractor on a separate sheet of paper.

- | | | |
|--------------------|---------------------|---------------------|
| 1. 43 degree angle | 2. 116 degree angle | 3. 135 degree angle |
| 4. 20 degree angle | 5. 165 degree angle | |

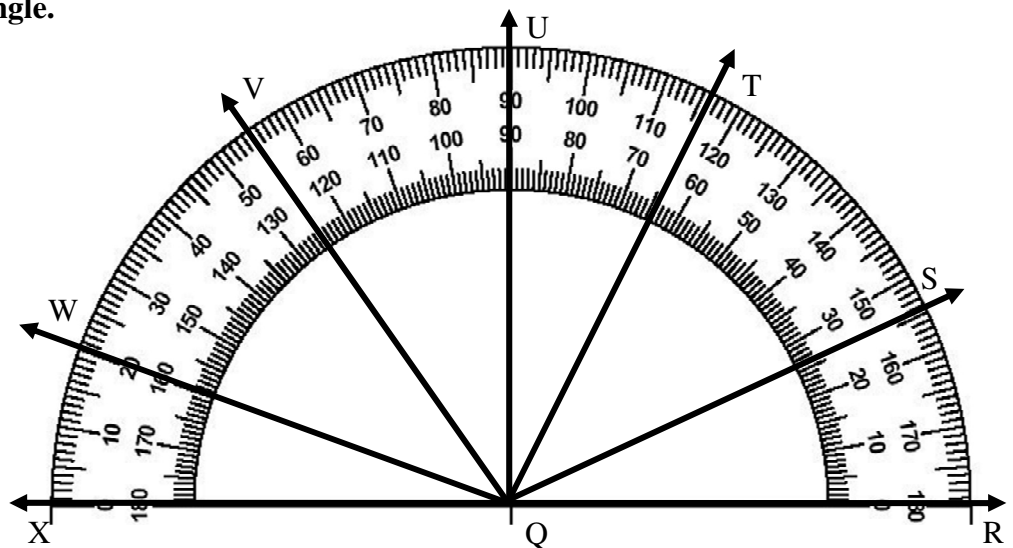
1. If you play golf, then you know the difference between a 3 iron and a 9 iron. Irons in the game of golf are numbered 1 to 10. The head of each is angled differently for different kinds of shots. The number 1 iron hits the ball farther and lower than a number 2, and so on. Use the table below to draw all the different golf club angles on the line segment below. Please use the 0 degree line as your starting point.

1 iron	15 degrees	6 iron	32 degrees
2 iron	18 degrees	7 iron	36 degrees
3 iron	21 degrees	8 iron	40 degrees
4 iron	25 degrees	9 iron	45 degrees
5 iron	28 degrees	Pitching wedge	50 degrees



Give the measure of each angle.

2.	$\angle RQS$	
3.	$\angle RQT$	
4.	$\angle RQU$	
5.	$\angle RQV$	
6.	$\angle RQW$	
7.	$\angle XQW$	
8.	$\angle XQT$	
9.	$\angle UQV$	
10.	$\angle VQT$	
11.	$\angle WQS$	



Complementary angles add up to 90° . **Supplementary angles** add up to 180° .

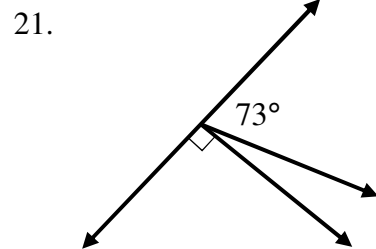
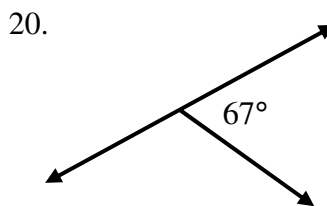
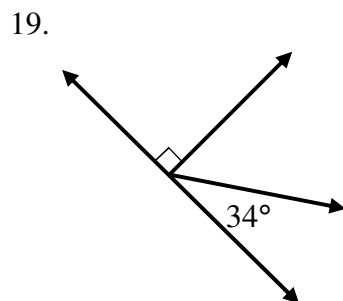
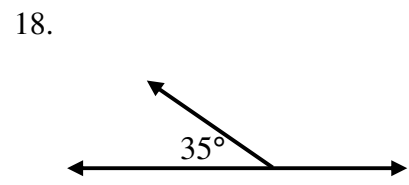
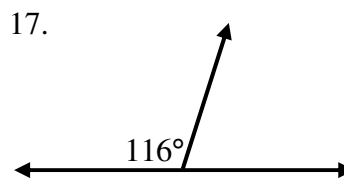
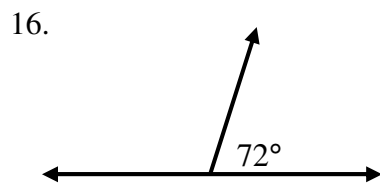
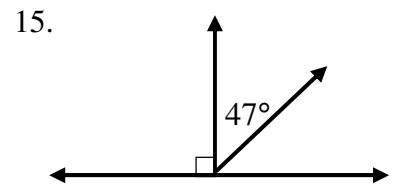
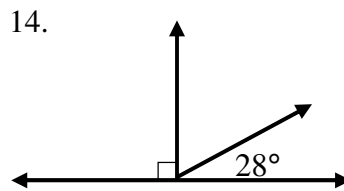
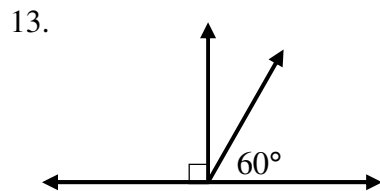
Find the measure of the angle that is complementary to the angle having the given measure.

1.	20°		2.	67°		3.	14°	
4.	81°		5.	45°		6.	74°	

Find the measure of the angle that is supplementary to the angle having the given measure.

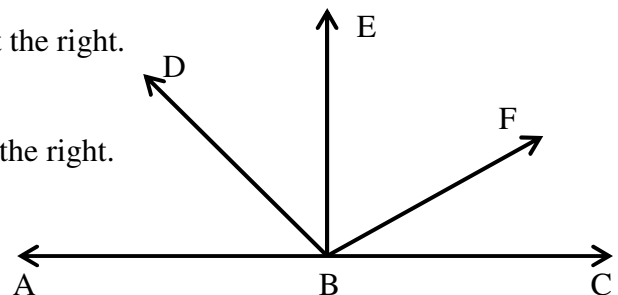
7.	120°		8.	56°		9.	29°	
10.	162°		11.	83°		12.	1°	

Find the angle measure that is not given.

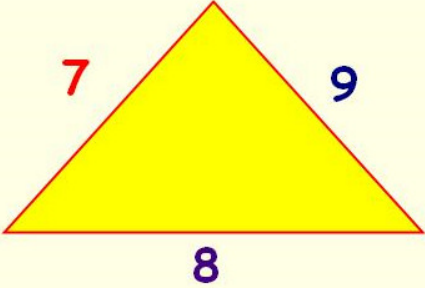


22. Name two complementary angles in the drawing at the right.

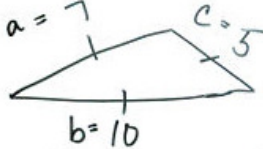
23. Name two supplementary angles in the drawing at the right.



Triangle Inequality Theorem



$7 + 8 > 9$
 $8 + 9 > 7$
 $9 + 7 > 8$



$1 = \text{True}$ $a + b > c$ $7 + 10 > 5$ $17 > 5$	$2 = \text{True}$ $a + c > b$ $7 + 5 > 10$ $12 > 10$	$3 = \text{True}$ $b + c > a$ $10 + 5 > 7$ $15 > 7$
--	---	--

Conclusion: Three sides
7, 10, 5 make a
Triangle

For any triangle, the sum of any two sides must be greater than the length of the third side.

Can a triangle be formed using the side lengths below? If so, classify the triangle as scalene, isosceles, or equilateral.

1.	5, 5, 5		2.	1, 6, 4	
3.	3, 2, 4		4.	6, 6, 4	
5.	1, 4, 1		6.	4, 4, 8	
7.	8, 6, 4		8.	3, 3, 7	
9.	7, 4, 4		10.	8, 4, 5	
11.	1, 2, 8		12.	12, 5, 13	

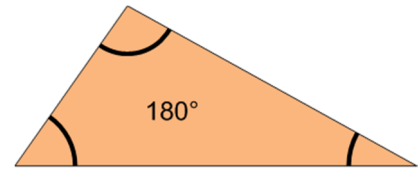
13.	Two sides of a triangle are 9 and 11 centimeters long. What is the shortest possible length in whole centimeters for the third side?	
14.	For the problem above what is the longest possible length?	

In each of the following you are given the length of two sides of a triangle. What can you conclude about the length of the third side?

15.	10 m, 8 m	16.	14 in, 20 in	17.	6 cm, 9 cm
18.	12 ft, 7 ft	19.	11 cm, 3 cm	20.	9 mm, 13 mm

Acute vision = sharp vision. Acute pain = a sharp pain.
 An acute angle between 0 and 90 degrees has a fairly sharp vertex.

Triangle Sum Theorem



The sum of the three angle measures in any triangle is always equal to 180°.

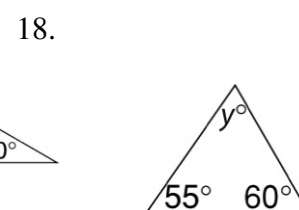
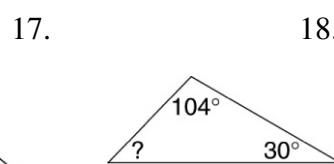
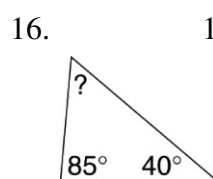
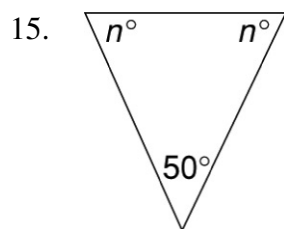
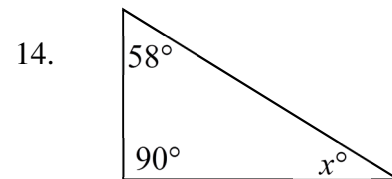
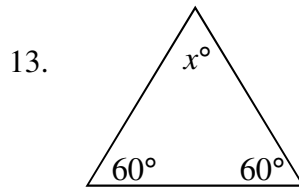
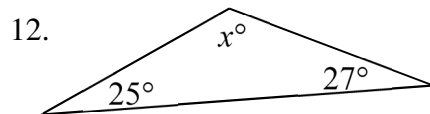
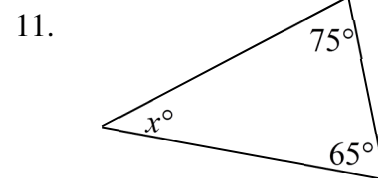
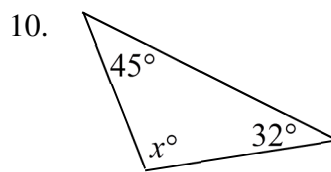
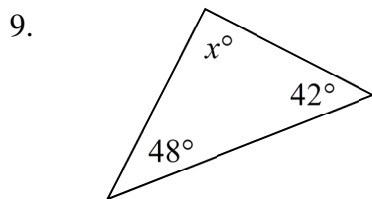
Classify the triangles as right, acute, or obtuse, given the three angles.

1.	40°, 30°, 110°		2.	60°, 30°, 90°	
3.	50°, 60°, 70°		4.	90°, 46°, 44°	

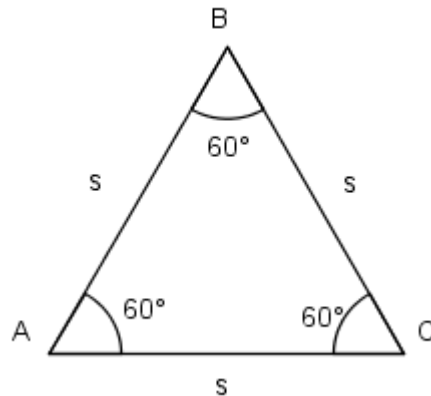
Classify each triangle as equilateral, isosceles, or scalene, given the lengths of the three sides.

5.	3 cm, 5 cm, 3 cm		6.	50 m, 50 m, 50 m	
7.	2 ft, 5 ft, 6 ft		8.	x mm, x mm, y mm	

Write and solve an equation to find the value of x . Then classify each triangle as acute, right, or obtuse.



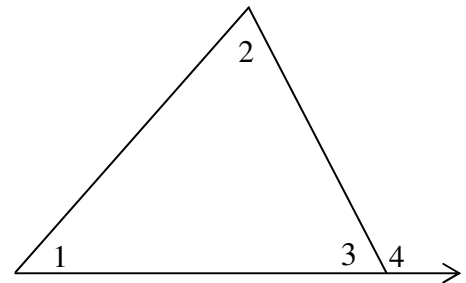
19.	The measures of the angles of a particular triangle are in a 5:6:7 ratio. What is the measure, in degrees, of the smallest angle?	
-----	---	--



The above triangle is equilateral. It is also an equiangular triangle since all angles are equal.

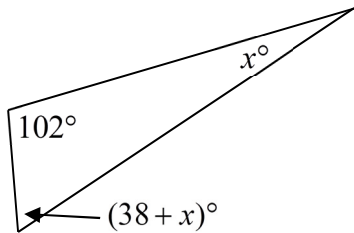
Use the figure at the right to solve each of the following.

1.	Find $m\angle 1$ if $m\angle 2 = 30^\circ$ and $m\angle 3 = 55^\circ$.	
2.	Find $m\angle 1$ if $m\angle 2 = 110^\circ$ and $m\angle 3 = 25^\circ$.	
3.	Find $m\angle 4$ if $m\angle 1 = 30^\circ$ and $m\angle 2 = 55^\circ$.	
4.	Find $m\angle 4$ if $m\angle 1 = 45^\circ$ and $m\angle 2 = 60^\circ$.	
5.	Find $m\angle 4$ if $m\angle 1 = 35^\circ$ and $m\angle 2 = 45^\circ$.	

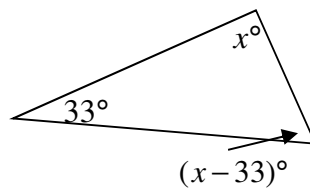


Using an equation, find x and then find the measure of the angles in each triangle.

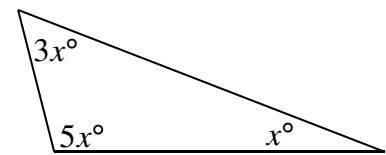
6.



7.

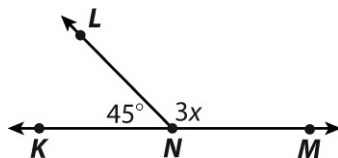


8.

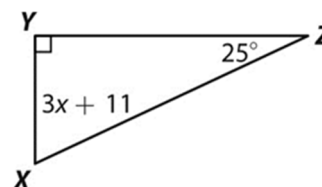


Using an equation, find x and then find the measure of the angles.

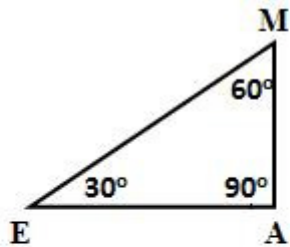
9.



10.



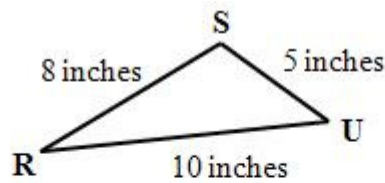
In a triangle, the side opposite the angle with the greatest measure is the longest side.



Since 90° is the largest angle of $\triangle AEM$, then \overline{ME} is the longest side.

Since 30° is the smallest angle of $\triangle AEM$, then \overline{MA} is the shortest side.

Since 60° is between 30° and 90° in $\triangle AEM$, then the length of \overline{EA} will be between the lengths of \overline{ME} and \overline{MA} .

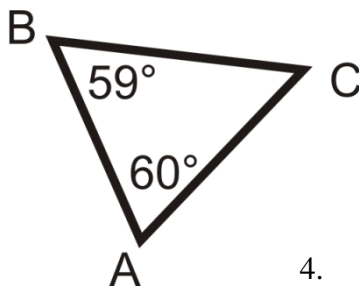


Since \overline{RU} is the longest side of $\triangle SUR$, then $\angle S$ is the largest angle.

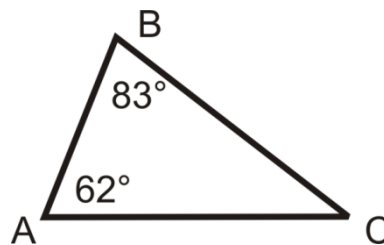
Since \overline{SU} is the shortest side of $\triangle SUR$, then $\angle R$ is the smallest angle.

Since the length of \overline{SR} is between the lengths of \overline{RU} and \overline{SU} in $\triangle SUR$, then the measure of $\angle U$ will be between the measures of $\angle S$ and $\angle R$.

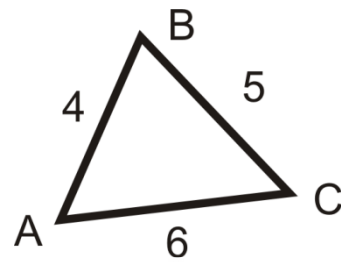
1.



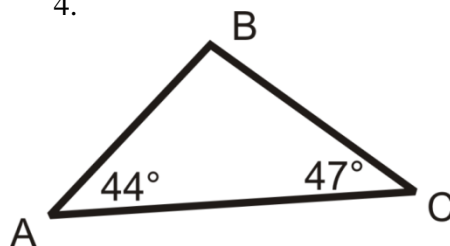
2.



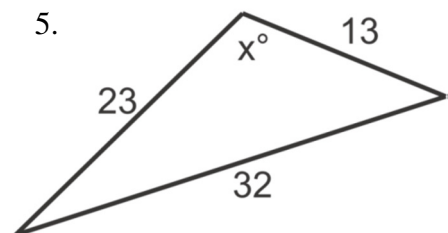
3.



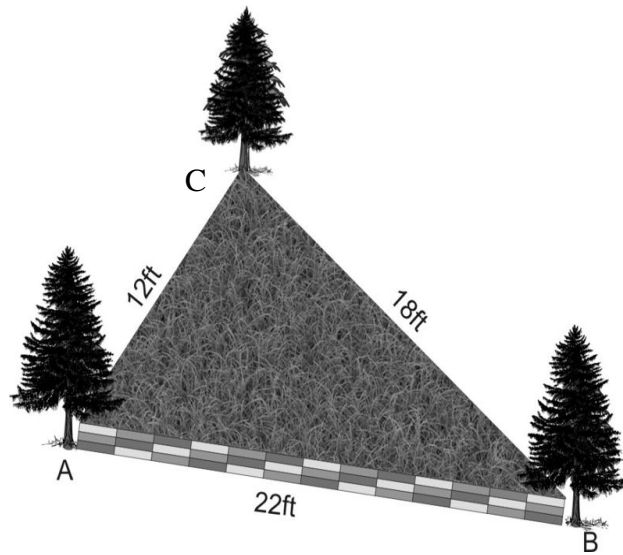
4.



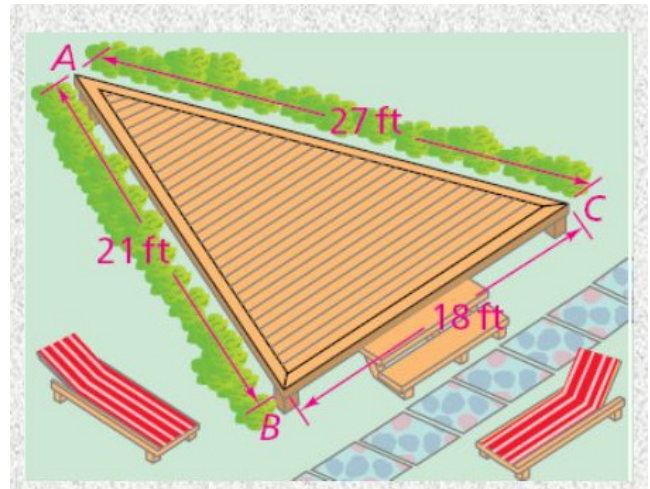
5.



1. Which angle is the second-largest angle?

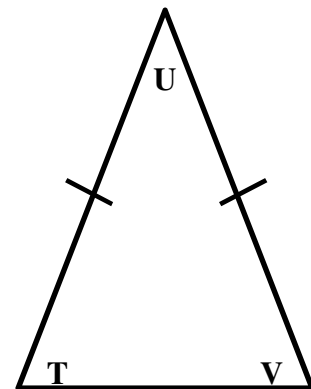


2. A landscaper wants to place benches in the two larger corners of the deck below. Which corners should she choose?



3. The measure of $\angle T$ in the triangle shown below is $2x$. Which can be used to find the value of x , the measure of $\angle U$?

- A. $2x^2 = 180$
- B. $2x + 2x + 2x = 180$
- C. $\frac{360}{2x} = 90$
- D. $2(2x) + x = 180$



Angle-Angle Criterion for Similarity

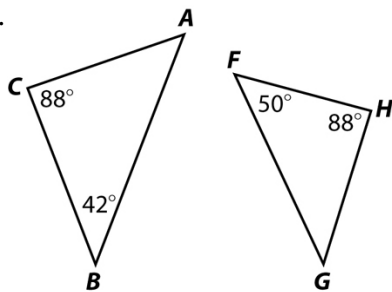
We know that the angles of a triangle must add up to 180° . This means that if a triangle has two angle measurements of 40° and 80° , then the third angle must be 60° . Now if a second triangle has two angle measurements of 40° and 60° , we know the third angle must be 80° . This means the two triangles are the same shape, but not necessarily the same size. Alternately we may think of one as a dilation of the other. Either way we know that the triangles are similar. We call this the **angle-angle criterion for similarity**.

Decide if the following triangles are similar and explain why using the angle-angle criterion.

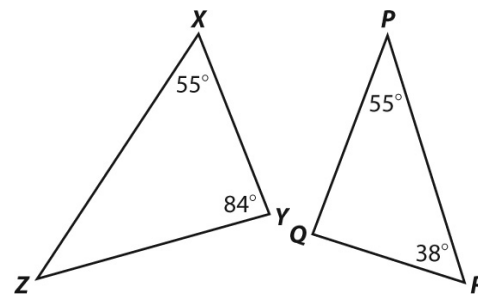
	Triangle 1	Triangle 2			Triangle 1	Triangle 2	
1.	$45^\circ, 45^\circ$	$45^\circ, 90^\circ$		2.	$50^\circ, 30^\circ$	$30^\circ, 100^\circ$	
3.	$60^\circ, 20^\circ$	$40^\circ, 100^\circ$		4.	$40^\circ, 30^\circ$	$90^\circ, 30^\circ$	
5.	$25^\circ, 115^\circ$	$25^\circ, 40^\circ$		6.	$5^\circ, 15^\circ$	$120^\circ, 15^\circ$	
7.	$5^\circ, 15^\circ$	$160^\circ, 15^\circ$		8.	$45^\circ, 55^\circ$	$55^\circ, 90^\circ$	
9.	$45^\circ, 30^\circ$	$30^\circ, 100^\circ$		10.	$80^\circ, 40^\circ$	$40^\circ, 60^\circ$	
11.	$105^\circ, 35^\circ$	$40^\circ, 105^\circ$		12.	$50^\circ, 50^\circ$	$50^\circ, 90^\circ$	
13.	$80^\circ, 30^\circ$	$70^\circ, 30^\circ$		14.	$72^\circ, 23^\circ$	$85^\circ, 23^\circ$	

Explain whether the triangles are similar.

15.

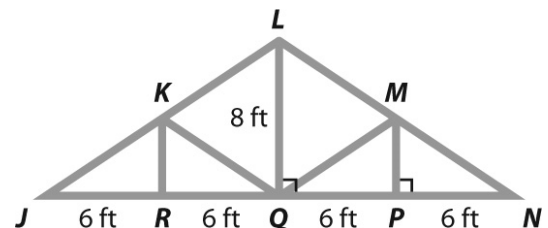


16.

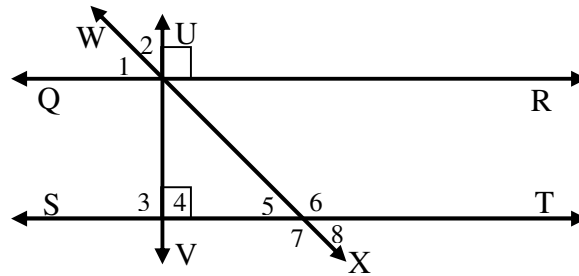


The diagram below shows a Howe roof truss, which is used to frame the roof of a building.

17.	Explain why $\triangle LQN$ is similar to $\triangle MPN$.	
18.	What is the length of support MP?	
19.	Using the information given in the diagram, can you determine whether $\triangle LQJ$ is similar to $\triangle KRJ$? Explain.	



Use the figure below to answer questions 1 through 7.

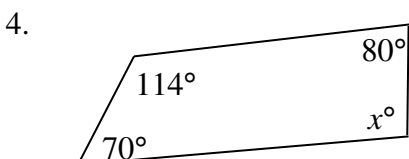
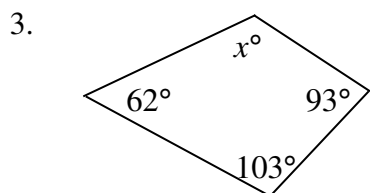
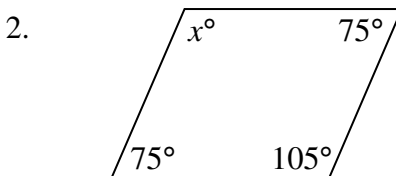
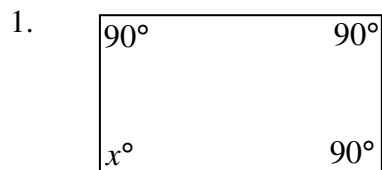


1.	Describe how QR and ST are related. A. They are perpendicular lines. B. They are parallel lines.	C. They are intersecting lines. D. They are complementary.	
2.	Describe how WX and UV are related. A. They are perpendicular lines. B. They are parallel lines.	C. They are intersecting lines. D. They are supplementary.	
3.	Describe how UV and ST are related. A. They are perpendicular lines. B. They are parallel lines.	C. They are complementary. D. They are right angles.	
4.	Which are complementary angles? A. $\angle 1$ and $\angle 2$ B. $\angle 5$ and $\angle 6$	C. $\angle 3$ and $\angle 4$ D. $\angle 7$ and $\angle 8$	
5.	Which are supplementary angles? A. $\angle 1$ and $\angle 2$ B. $\angle 5$ and $\angle 8$	C. $\angle 4$ and $\angle 5$ D. $\angle 7$ and $\angle 8$	
6.	If the measure of $\angle 5$ is 45° , what is the measure of $\angle 6$?		
7.	What is the measure of $\angle 3$?		
8.	What is the measure of x in the parallelogram?		

Write an equation to find x . Then find the measure of the missing angles in each triangle.

	Angle 1	Angle 2	Angle 3	Angle 1	Angle 2	Angle 3
9.	x°	$x + 20^\circ$	$x + 70^\circ$			
10.	$2x - 40^\circ$	$x + 10^\circ$	$3x - 60^\circ$			
11.	x°	$120 - x^\circ$	$100 - x^\circ$			
12.	$3x^\circ$	$2x^\circ$	$5x^\circ$			

Find the value of x .



Write an equation to find x and then find all the missing angles.

5.	A trapezoid with angles 115° , 65° , 55° , and x° .	
6.	A quadrilateral with angles 104° , 60° , 140° , and x° .	
7.	A parallelogram with angles 70° , 110° , $(x+40)^\circ$, and x° .	
8.	A quadrilateral with angles x° , $2x^\circ$, $3x^\circ$, and $4x^\circ$.	
9.	A quadrilateral with angles $(x+30)^\circ$, $(x-55)^\circ$, x° , and $(x-45)^\circ$.	
10.	Which of the following could be the angle measures in a parallelogram (all numbers are in degrees): a) 19, 84, 84, 173 b) 24, 92, 92, 152 c) 33, 79, 102, 146 d) 49, 49, 131, 131	

For any polygon with n sides, the following formula can be used to calculate the sum of the angles:

$$180(n - 2)$$

Find the sum of the measures of the angles of each polygon.

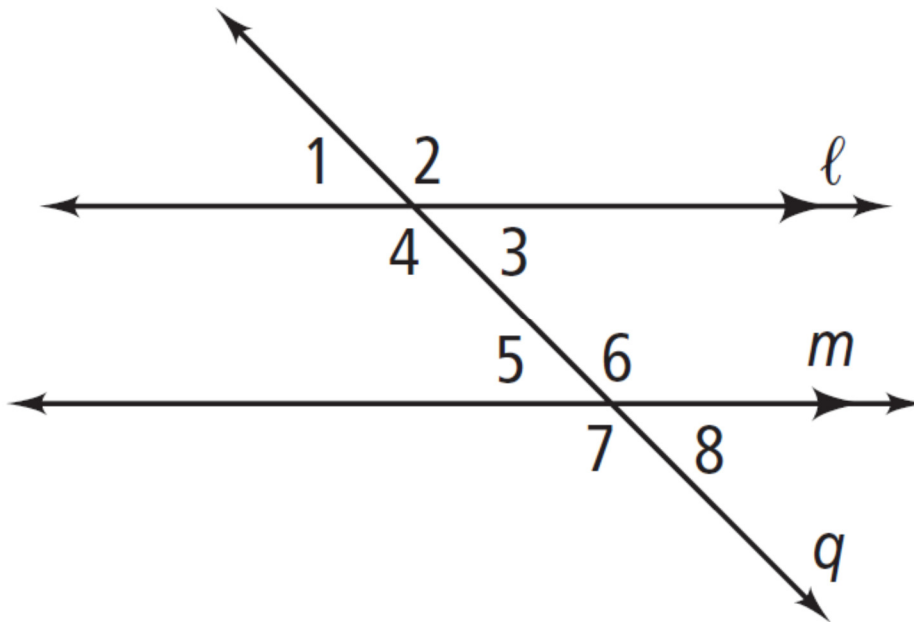
11.	quadrilateral		12.	pentagon		13.	octagon	
14.	12-gon		15.	18-gon		16.	30-gon	
17.	75-gon		18.	100-gon				

For any polygon with n sides, the following formula can be used to calculate the average angle of size:

$$\frac{180(n - 2)}{n}$$

Find the measure of each angle of each regular polygon (nearest tenth).

19.	regular octagon		20.	regular pentagon	
21.	regular heptagon		22.	regular nonagon	
23.	regular 18-gon		24.	regular 25-gon	



Find the two **parallel lines** on the diagram above. Trace the **parallel lines** in *green color pencil*.

1.	Which two lines are parallel ?	
----	---------------------------------------	--

Find the **transversal** on the diagram above. Label the **transversal** and trace it in *red color pencil*.

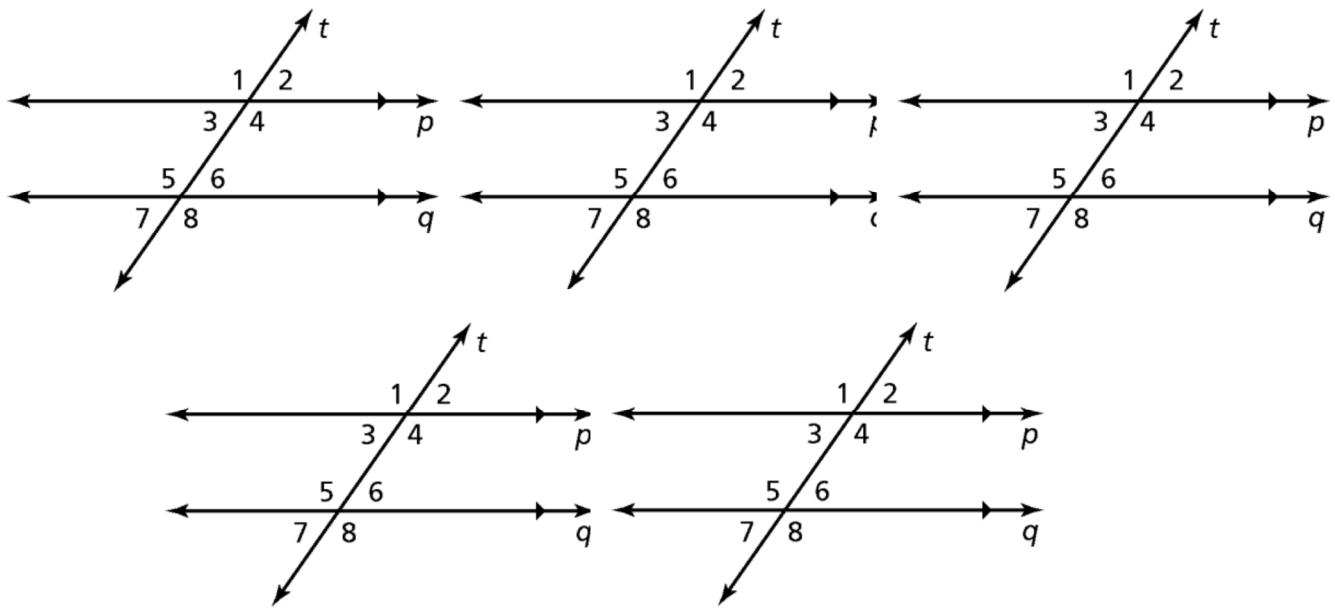
2.	Which line is the transversal ?	
----	--	--

Find the **interior region** on the diagram above. Label the **interior region** on the diagram in *purple color pencil*.

3.	Which four angles are in the interior region ? Color these angles in <i>purple color pencil</i> .	
----	---	--

Find the **exterior region** on the diagram above. Label the **exterior region** on the diagram in *blue color pencil*.

4.	Which angles are in the exterior region ? Color these angles in <i>blue color pencil</i> .	
----	---	--



Find a pair of **corresponding angles** and mark the angles in the diagram in *blue color pencil*.

5.	Name of your two corresponding angles. What is special about the angle measures of corresponding angles?	
----	--	--

Find a pair of **alternate interior angles** and mark the angles in the diagram in *green color pencil*.

6.	Name of your two alternate interior angles. What is special about the angle measures of alternate interior angles?	
----	--	--

Find a pair of **alternate exterior angles** and mark the angles in the diagram in *orange color pencil*.

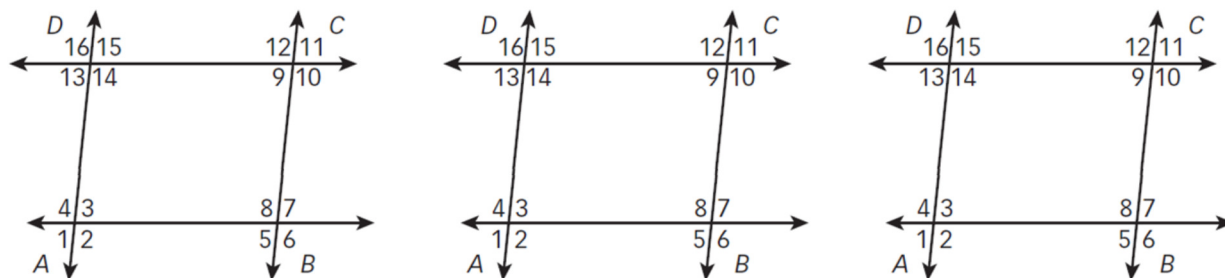
7.	Name of your two alternate exterior angles. What is special about the angle measures of alternate exterior angles?	
----	--	--

Find a pair of **same-side interior angles** and mark the angles in the diagram in *red color pencil*.

8.	Name of your two same-side interior angles. What is special about the angle measures of same-side interior angles?	
----	--	--

Find a pair of **same-side exterior angles** and mark the angles in the diagram in *purple color pencil*.

9.	Name of your two same-side exterior angles. What is special about the angle measures of same-side exterior angles?	
----	--	--



$\overline{AD} \parallel \overline{BC}$ and $\overline{DC} \parallel \overline{AB}$. In red color pencil, put parallel line arrows on the lines that are parallel.

Find two angles that are **congruent** to $\angle 1$ and mark the angles in red color pencil.

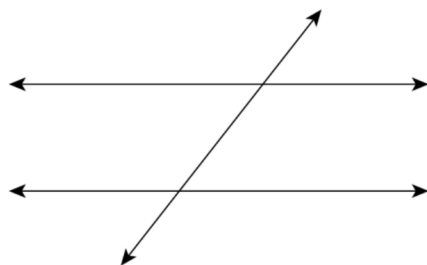
Angle	Reason	Angle	Reason

Find two angles that are **supplementary** to $\angle 8$ and mark the angles in blue color pencil.

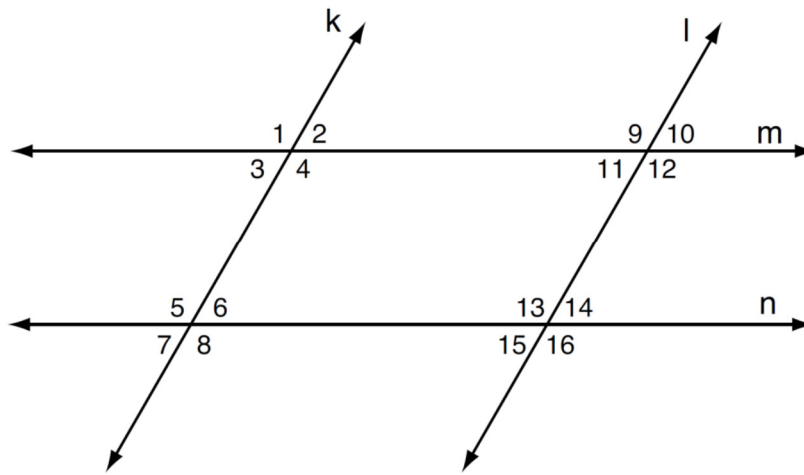
Angle	Reason	Angle	Reason

Find four angles that are **congruent** to $\angle 9$ and mark the angles in green color pencil.

Angle	Reason	Angle	Reason

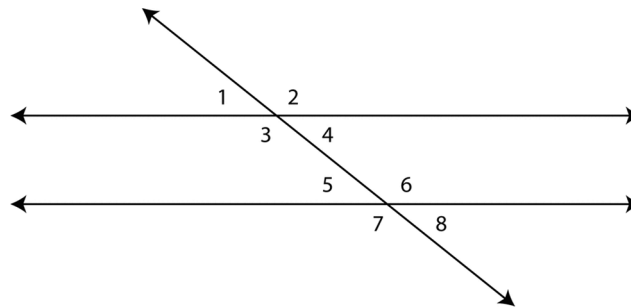


10.	If one of the angles formed when a transversal intersected two parallel lines measures 38° , how many other angles would measure 38° ?	
11.	If a transversal intersects two parallel lines and four angles have a measure of 72° , then what is the measure of the other four angles?	
12.	Two lines are cut by a transversal so that $\angle 1$ and $\angle 3$ are same-side interior angles and $\angle 1$ and $\angle 4$ are alternate interior angles. If $m\angle 3 = 48^\circ$, then what is the measure of angle 4?	



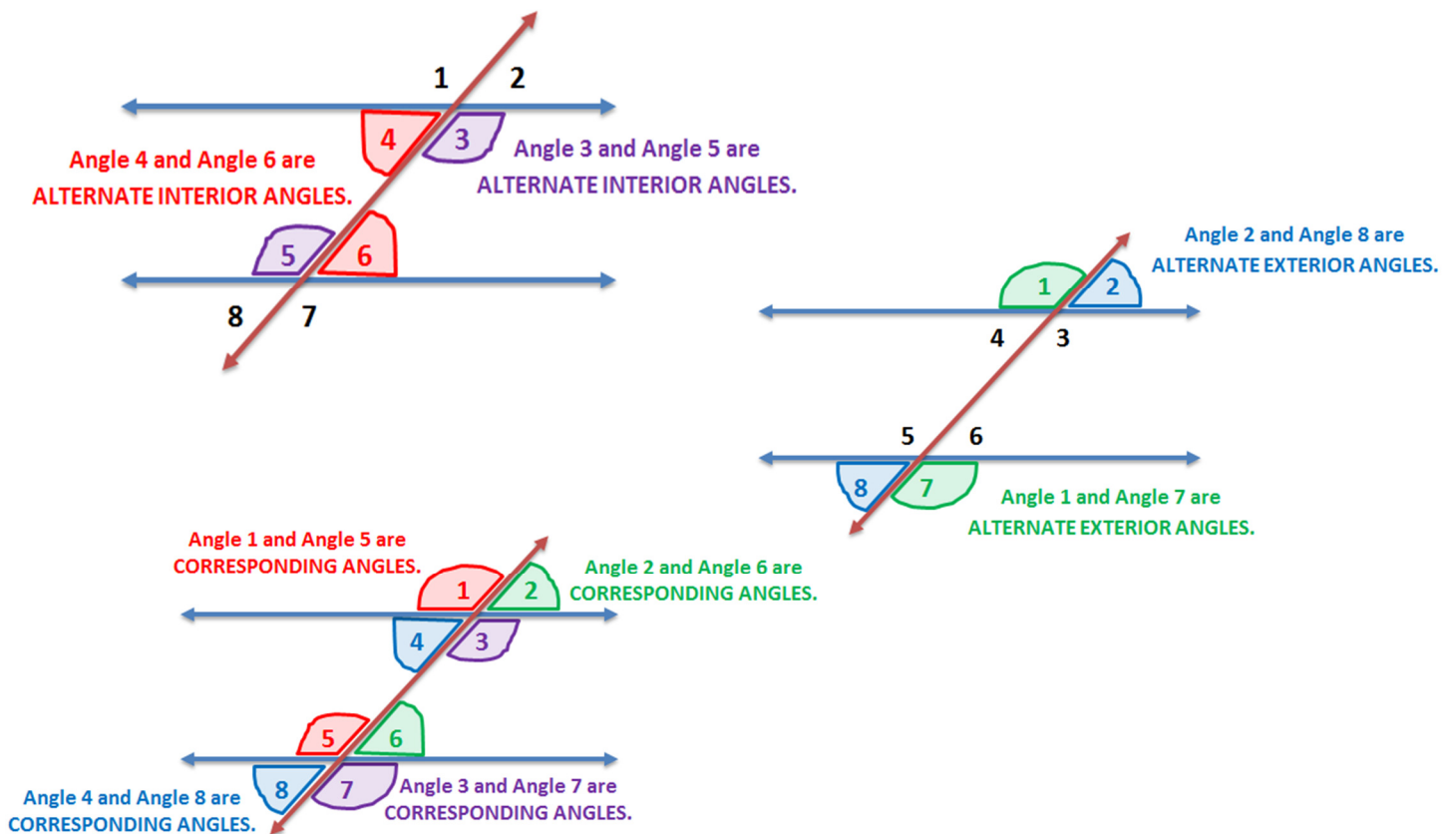
1.	If $m\angle 1 = 135^\circ$, $m\angle 2 =$ _____.	Angle relationship:
2.	If $m\angle 3 = 62^\circ$, $m\angle 6 =$ _____.	Angle relationship:
3.	If $m\angle 13 = 130^\circ$, $m\angle 16 =$ _____.	Angle relationship:
4.	If $m\angle 9 = 110^\circ$, $m\angle 13 =$ _____.	Angle relationship:
5.	If $m\angle 5 = 115^\circ$, $m\angle 16 =$ _____.	Angle relationship:
6.	If $m\angle 10 = 60^\circ$, $m\angle 15 =$ _____.	Angle relationship:
7.	If $m\angle 4 = 105^\circ$, $m\angle 5 =$ _____.	Angle relationship:
8.	If $m\angle 13 = 105^\circ$, $m\angle 6 =$ _____.	Angle relationship:
9.	If $m\angle 11 = 65^\circ$, $m\angle 14 =$ _____.	Angle relationship:
10.	If $m\angle 9 = 125^\circ$, $m\angle 15 =$ _____.	Angle relationship:
11.	If $m\angle 10 = 52^\circ$, $m\angle 3 =$ _____.	Angle relationship:
12.	If $m\angle 7 = 83^\circ$, $m\angle 15 =$ _____.	Angle relationship:
13.	If $m\angle 8 = 113^\circ$, $m\angle 13 =$ _____.	Angle relationship:
14.	If $m\angle 12 = 140^\circ$, $m\angle 4 =$ _____.	Angle relationship:
15.	If $m\angle 16 = 100^\circ$, $m\angle 14 =$ _____.	Angle relationship:
16.	If $m\angle 9 = 100^\circ$, $m\angle 12 =$ _____.	Angle relationship:
17.	If $m\angle 6 = 68^\circ$, $m\angle 4 =$ _____.	Angle relationship:

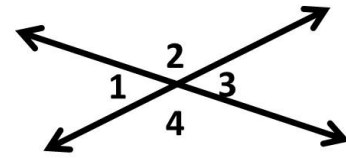
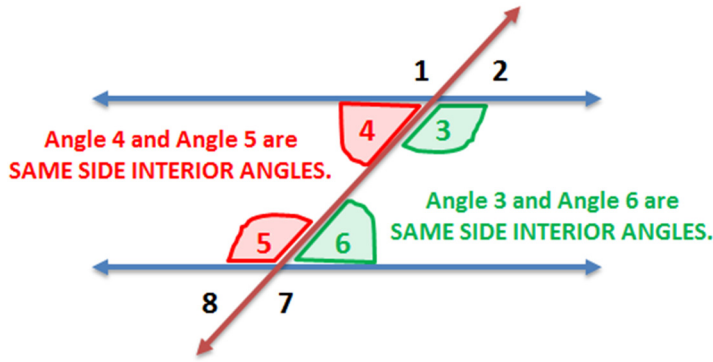
A **transversal** is a line that intersects two or more other lines to form eight or more angles.



1.	Name three pairs of angles above that are supplementary.	
2.	Which angles appear to be acute?	
3.	Which angles appear to be obtuse?	
4.	If $\angle 1 = (x + 25)^\circ$ and $\angle 2 = 85^\circ$ find the size of all the other listed angles.	
5.	If $\angle 1 = 5x^\circ$ and $\angle 2 = 65^\circ$ find the size of all the other listed angles.	

Alternate angles are on opposite sides of the transversal and have a different vertex. There are two pairs of angles in the diagram that are referred to as **alternate exterior angles** and two pairs of angles that are referred to as **alternate interior angles**.





Angles 1 and 3 and Angles 2 and 4 are pairs of vertical angles.

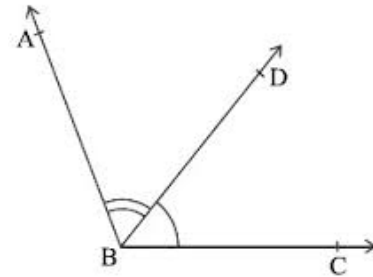
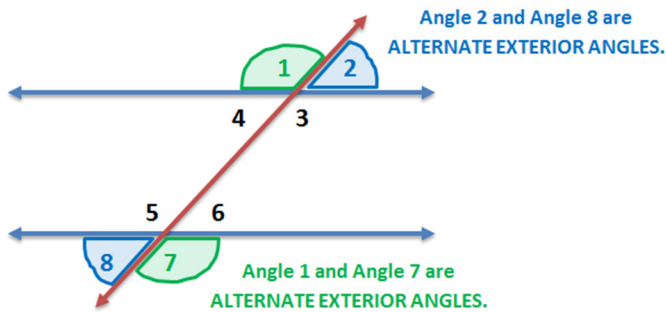
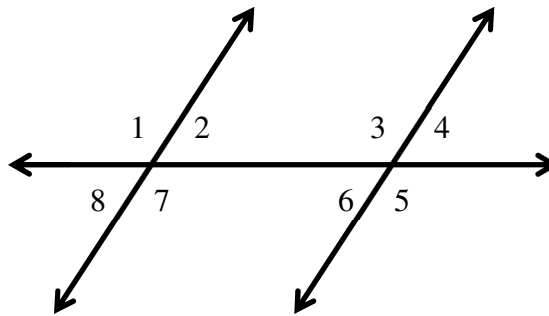
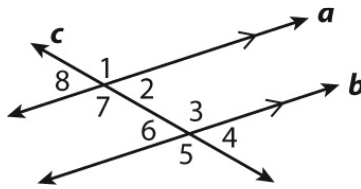


Fig. 2 Adjacent angles



1.	Name all pairs of vertical angles in the figure.	
2.	Name all pairs of alternate interior angles in the figure.	
3.	Name all pairs of alternate exterior angles in the figure.	
4.	Name all pairs of corresponding angles in the figure.	
5.	Name two pairs of adjacent angles.	
6.	Name all of the angles that are supplementary to $\angle 8$.	
7.	If $m\angle 2 = 57^\circ$, find $m\angle 3$ and $m\angle 4$.	
8.	If $m\angle 6 = (5x + 1)^\circ$ and $m\angle 8 = (7x - 23)^\circ$, find $m\angle 6$ and $m\angle 8$.	
9.	Suppose $\angle 9$, which is not shown in the figure, is complementary to $\angle 4$. Given that $m\angle 1 = 153^\circ$, what is $m\angle 9$?	

Use the figure below for Exercises 1–6.



1.	Name both pairs of alternate interior angles.	
2.	Name the corresponding angle to $\angle 3$.	
3.	Name the relationship between $\angle 1$ and $\angle 5$.	
4.	Name the relationship between $\angle 2$ and $\angle 3$.	
5.	Name the interior angles that are supplementary to $\angle 7$.	
6.	Name the exterior angles that are supplementary to $\angle 5$.	

Use the figure at the right for problems 7–10. Line $MP \parallel$ line QS .

Find the angle measures.

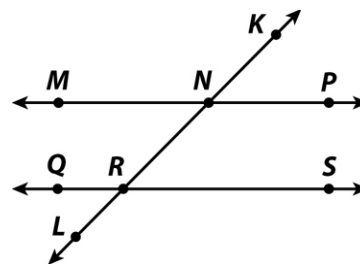
7. $m\angle KRQ$ when $m\angle KNM = 146^\circ$ _____

8. $m\angle QRN$ when $m\angle MNR = 52^\circ$ _____

If $m\angle RNP = (8x + 63)^\circ$ and $m\angle NRS = 5x^\circ$, find the following angle measures.

9. $m\angle RNP =$ _____

10. $m\angle NRS =$ _____

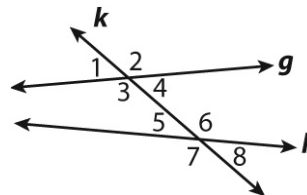


In the figure at the right, there are no parallel lines. Use the figure for problems 11–14.

11. Name both pairs of alternate exterior angles.

12. Name the corresponding angle to $\angle 4$ _____

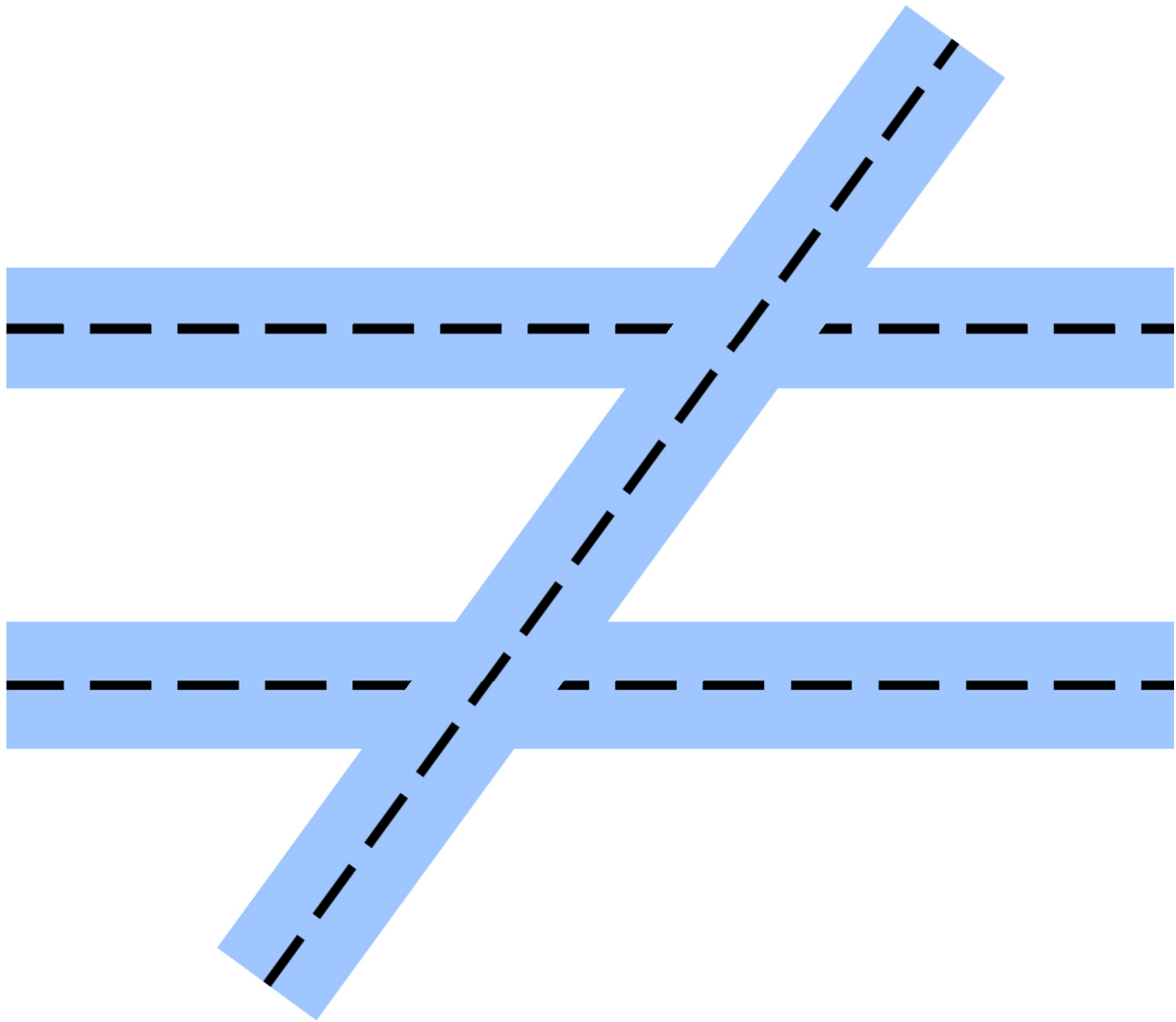
13. Name the relationship between $\angle 3$ and $\angle 6$.



14. Are there any supplementary angles? If so, name two pairs. If not, explain why not.

Using the roads below, you are to create a map of the road crossing in Southlake. You have 8 total street corners. Place the following locations at one street corner on the diagram below. Please use color.

Your house	Target	Bank of America	Post Office
Starbucks	Hospital	Chick-Fil-A	Park



Next, on the back of this sheet, use the vocabulary words provided to describe the locations of your buildings. You must use all of the words exactly twice. That means that you will have a total of **12 sentences**. For example: The bank and the post office are alternate interior angles.

Alternate Interior	Same Side Interior	Alternate Exterior	Corresponding
Supplementary		Vertical	

Complete the sentences below.

1.		and		are	
2.		and		are	
3.		and		are	
4.		and		are	
5.		and		are	
6.		and		are	
7.		and		are	
8.		and		are	
9.		and		are	
10.		and		are	
11.		and		are	
12.		and		are	

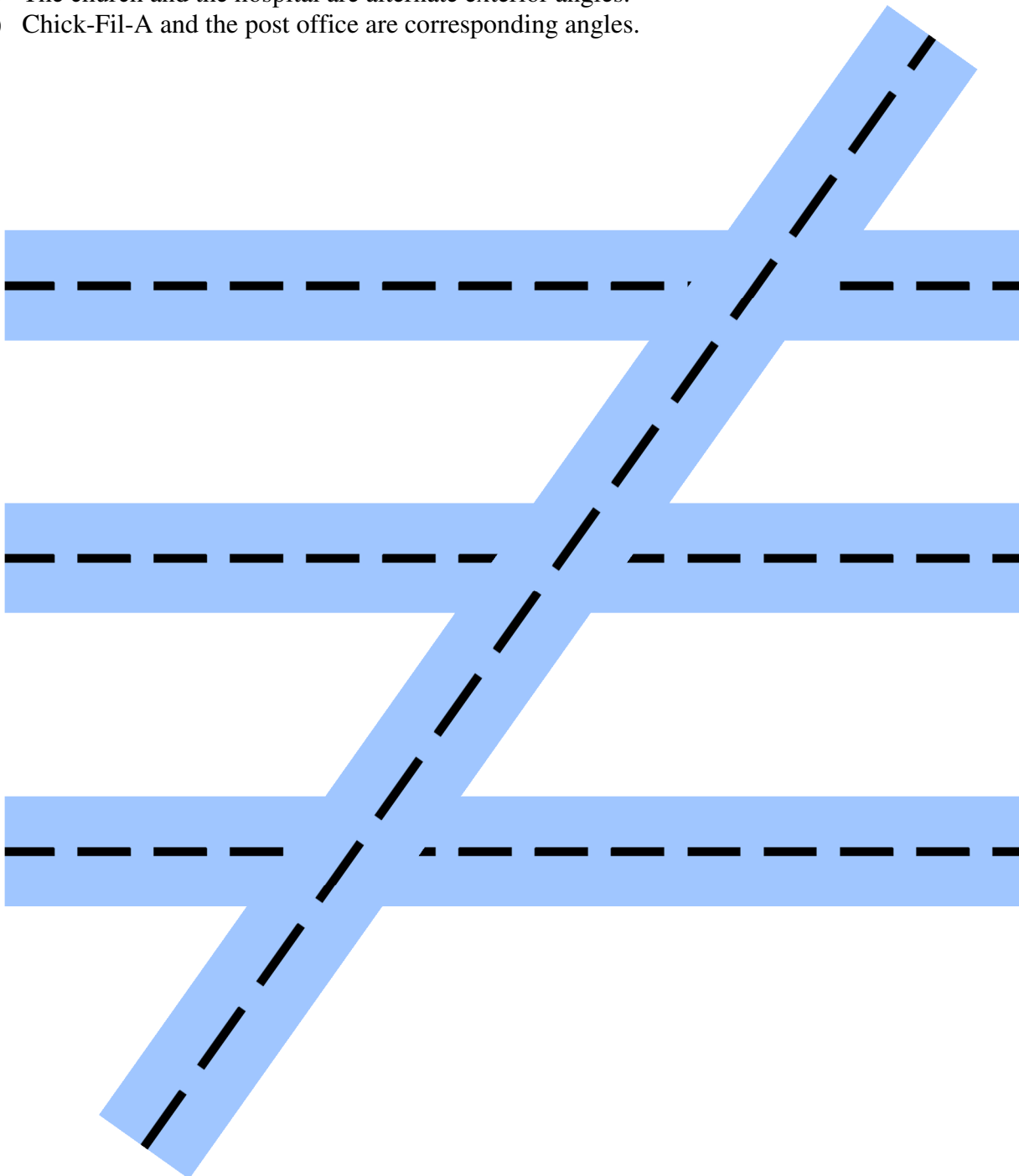
Activity 9-45: **Parallel Lines Cut by a Transversal**

Name: _____

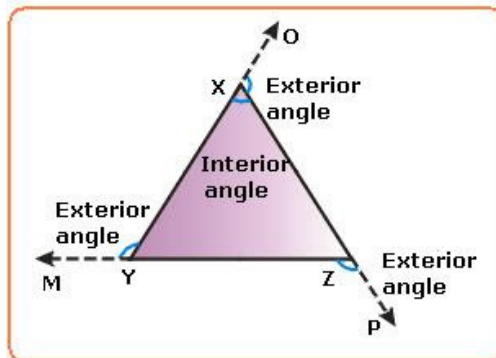
Using the picture below, you are to create a new map of Southlake. Draw the buildings on the appropriate corners using the directions below. Make it neat, label it, and be creative.

Construction:

- 1) The Bank of America and Target are alternate interior angles.
- 2) Exxon and the post office are vertical angles.
- 3) Your house and the post office are same side interior angles.
- 4) Your house and the park are alternate interior angles.
- 5) Exxon and the hospital are a pair of supplementary angles.
- 6) The church and the hospital are alternate exterior angles.
- 7) Chick-Fil-A and the post office are corresponding angles.

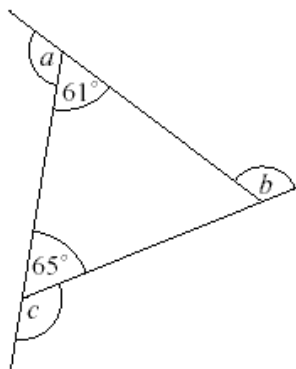


An **exterior angle** of a triangle is formed by extending a side of the triangle.



Describe the relationship between $\angle ZYX$ and $\angle XYM$.

Find the measure of each of the three exterior angles.



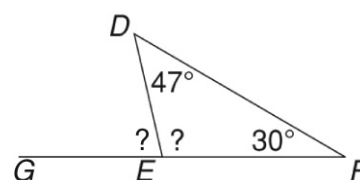
1. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles**. Complete the table below.

Exterior Angle	Exterior Angle Size	Size of each remote interior angle	Sum of the two remote interior angles
$\angle A$			
$\angle B$			
$\angle C$			

Use the diagram at the right to answer each question below.

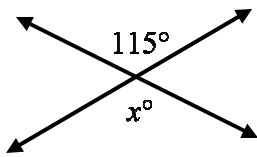
2. What is the measure of $\angle DEF$?

3. What is the measure of $\angle DEG$?

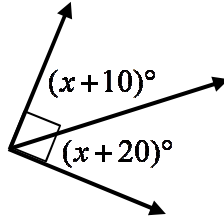


Find x and the measure of each unknown angle by writing and solving an equation.

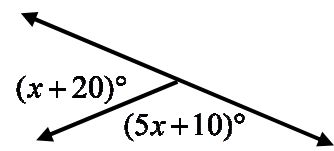
1.



2.



3.



In the figure at the right $m \parallel n$. Find the measure of each angle.

3.	$\angle 1$		4.	$\angle 2$	
5.	$\angle 3$		6.	$\angle 4$	
7.	$\angle 5$		8.	$\angle 6$	
9.	$\angle 7$		10.	$\angle 8$	
11.	$\angle 9$		12.	$\angle 10$	

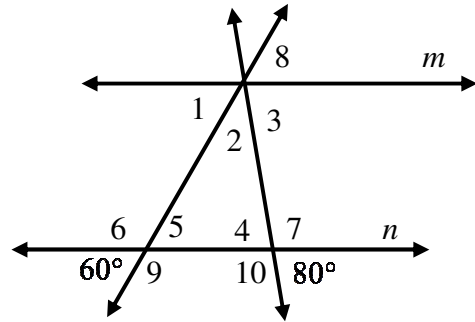
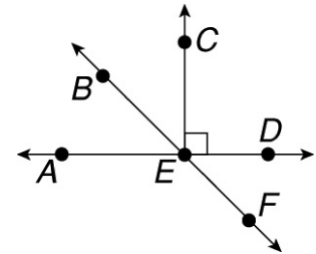


	Figure 1 	Figure 2 	
13.	Use Figure 1 to find the following: Find x . Find the measure of $\angle ABE$.		
14.	Use Figure 2 to find the following: Find x . Find the measure of $\angle ABE$.		

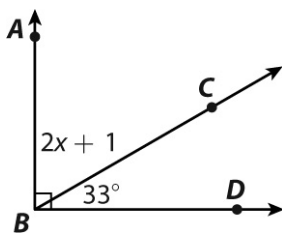
Use the figure at the right.

15.	Name a pair of vertical angles. Name a pair of complementary angles. Name a pair of supplementary angles.



Find the value of x in each figure by writing and solving an equation.

16.



17.

